Section 8.1 연습문제

1.

풀이 $$\int_0^2\int_{-1}^2 (x+y+2)\,dx\,dy = \int_0^2\left(\frac{1}{2}x^2+(y+2)x\,\Big|_{x=-1}^{x=2}\right)dy = \int_0^2\left(\frac{15}{2}+3y\right)dy$$

$$= \frac{15}{2}y+\frac{3}{2}y^2\,\Big|_0^2 = 21$$

3.

풀이 $$\int_0^{\pi/2}\int_{-\pi}^{\pi} (\sin x+\cos y)\,dx\,dy = \int_0^{\pi/2}\left(-\cos x + x\cos y\,\Big|_{x=-\pi}^{x=\pi}\right)dy$$

$$= 2\pi\int_0^{\pi/2}\cos y\,dy = 2\pi\,\sin y\,\Big|_0^{\pi/2} = 2\pi$$

5.

풀이 $$\int_0^{\ln 2}\int_0^{\ln y} e^{2x+y}\,dx\,dy = \int_0^{\ln 2}\left(\frac{1}{2}e^{2x+y}\,\Big|_{x=0}^{x=\ln y}\right)dy = \frac{1}{2}\int_0^{\ln 2}(y^2-1)e^y\,dy$$

$$= \frac{1}{2}(y-1)^2e^y\,\Big|_0^{\ln 2} = -\frac{1}{2}+(-1+\ln 2)^2$$

7.

풀이 $$\int_0^2\int_0^1 \frac{1}{(x+1)(y+1)}\,dy\,dx = \int_0^2\left(\frac{\ln(y+1)}{1+x}\,\Big|_{y=0}^{1}\right)dx = \int_0^2\frac{\ln 2}{1+x}\,dx$$

$$= \ln 2\left(\ln(x+1)\,\Big|_0^2\right) = (\ln 2)(\ln 3)$$

9.

풀이 $$\int_0^{\pi/2}\int_0^{\cos y} e^{\sin y}\,dx\,dy = \int_0^{\pi/2}\int_0^{\cos y}\left(x\,e^{\sin y}\,\Big|_{x=0}^{x=\cos y}\right)dy = \int_0^{\pi/2}\cos y\,e^{\sin y}\,dy$$

$$= e^{\sin y}\,\Big|_0^{\pi/2} = e-1$$

11.

풀이 $\displaystyle\int_0^1\int_0^y \sqrt{1-x^2}\,dx\,dy = \frac{1}{2}\int_0^1\left(x\sqrt{1-x^2}+\sin^{-1}x\,\Big|_{x=0}^{x=y}\right)dy$

$$= \frac{1}{2}\int_0^1\left(y\sqrt{1-y^2}+\sin^{-1}y\right)dy$$

$$= \frac{1}{2}\left(\sqrt{1-y^2}-\frac{1}{3}(1-y^2)^{3/2}+y\sin^{-1}y\right)\Bigg|_0^1 = \frac{3\pi-4}{12}$$

13.

풀이 $\displaystyle\int_0^1\int_{\sqrt{y}}^1 \frac{\sin x}{x}\,dx\,dy = \int_0^1\int_0^{x^2}\frac{\sin x}{x}\,dy\,dx = \int_0^1\left(\frac{y\sin x}{x}dx\,\Bigg|_0^{x^2}\right)dx$

$$= \int_0^1 x\sin x\,dx = -x\cos x+\sin x\,\Big|_0^1 = \sin 1-\cos 1$$

15.

풀이 $\displaystyle\int_0^4\int_{x/2}^2 e^{y^2}\,dy\,dx = \int_0^2\int_0^{2y}e^{y^2}\,dx\,dy = \int_0^2\left(x\,e^{y^2}\,\Big|_{x=0}^{x=2y}\right)dy$

$$= \int_0^2 2y\,e^{y^2}\,dy = e^{y^2}\,\Big|_0^2 = e^4-1$$

17.

풀이 $\displaystyle\int_0^2\int_0^{4-x^2}\frac{x\,e^{2y}}{4-y}\,dy\,dx = \int_0^4\int_0^{\sqrt{4-y}}\frac{x\,e^{2y}}{4-y}\,dx\,dy = \int_0^4\left(\frac{x^2e^{2y}}{2(4-y)}\Bigg|_0^{\sqrt{4-y}}\right)dx$

$$= \frac{1}{2}\int_0^4 e^{2y}dy = \frac{e^{2y}}{4}\Bigg|_0^4 = \frac{e^8-1}{4}$$

19.

풀이 $$\int_0^1\int_0^{\cos^{-1}y}\sin x\sqrt{1+\sin^2 x}\,dx\,dy=\int_0^{\pi/2}\int_0^{\cos x}\sin x\sqrt{1+\sin^2 x}\,dy\,dx$$
$$=\int_0^{\pi/2}\left(y\sin x\sqrt{1+\sin^2 x}\,\Big|_0^{\cos x}\right)dx$$
$$=\int_0^{\pi/2}\sin x\sqrt{1+\sin^2 x}\,\cos x\,dx$$
$$=\int_0^1 t\sqrt{1+t^2}\,dt=\frac{-1+2\sqrt{2}}{3}$$

21.

풀이 교점의 y좌표 : $y=\pm 2$, $y=0$

$$A=\iint_D dx\,dy=\int_{-2}^{0}\int_{y^2-y^3}^{y^2-4y}dx\,dy+\int_0^2\int_{y^2-4y}^{y^2-y^3}dx\,dy=4+4=8$$

23.

풀이 교점의 y좌표 : $y=0$, $y=3$

$$A=\iint_D dx\,dy=\int_0^3\int_{y^2-4y}^{2y-y^2}dx\,dy=9$$

25.

풀이 푸비니 정리에 의하여 적분순서를 바꾸면 다음과 같다.

$$\iint_D f(x,y)\,dV=\int_0^1\int_0^3\frac{x^2}{(y-1)^{2/3}}\,dy\,dx=\int_0^3\int_0^1\frac{x^2}{(y-1)^{2/3}}\,dx\,dy$$
$$=\int_0^3\frac{1}{(y-1)^{2/3}}\left(\frac{1}{3}x^3\,\Big|_0^1\right)dy=\frac{1}{3}\int_0^3\frac{dy}{(y-1)^{2/3}}$$

이고

$$\frac{1}{3}\int_0^3 \frac{dy}{(y-1)^{2/3}} = \frac{1}{3}\int_0^1 \frac{dy}{(y-1)^{2/3}} + \frac{1}{3}\int_1^3 \frac{dy}{(y-1)^{2/3}}$$

$$= \frac{1}{3}\lim_{a\to 1-}\int_0^a \frac{dy}{(y-1)^{2/3}} + \frac{1}{3}\lim_{a\to 1+}\int_a^3 \frac{dy}{(y-1)^{2/3}}$$

$$= \lim_{a\to 1-}\left((a-1)^{1/3}+1\right) - \lim_{a\to 1+}\left((a-1)^{1/3}-\sqrt[3]{2}\right) = 1+\sqrt[3]{2}$$

이므로 $\iint_D f(x,\, y)\,dV = 1+\sqrt[3]{2}$ 이다.

27.

풀이 $A_1 = \int_{-1}^0 \int_0^{(x+1)^2} dy\,dx = 2\int_{-1}^0 (1+x)^2\,dx = \frac{2}{3}$

$A_2 = \int_0^1 \int_0^{y-y^3} dx\,dy = 2\int_0^1 (y-y^3)\,dy = \frac{1}{2}$

$A_3 = \int_{-1}^0 \int_{-1}^{y-y^3} dx\,dy = 2\int_{-1}^0 (1+y-y^3)\,dy = \frac{3}{2}$

$A = A_1 + A_2 + A_3 = \frac{8}{3}$

29.

풀이 $\int_0^{\pi/4}\int_{\sin x}^{\cos x} dy\,dx = \int_0^{\pi/4} (\cos x - \sin x)\,dx = \sqrt{2}-1$

$\int_{\pi/4}^{\pi/2}\int_{\cos x}^{\sin x} dy\,dx = \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx = \sqrt{2}-1,\quad A = A_1 + A_2 = 2(\sqrt{2}-1)$

Section 8.2 연습문제

1.

풀이 $$\int_{-2}^{2}\int_{0}^{3}\int_{-1}^{2} 2\,dz\,dy\,dx = \int_{-2}^{2}\int_{0}^{3}\left(2z\Big|_{z=-1}^{z=2}\right)dy\,dx = \int_{-2}^{2}\int_{0}^{3} 6\,dy\,dx$$

$$= \int_{-2}^{2}\left(6y\Big|_{y=0}^{y=3}\right)dx = \int_{-2}^{2} 18dx = 18x\Big|_{-2}^{2} = 72$$

3.

풀이 $$\int_{0}^{\pi}\int_{0}^{\pi}\int_{0}^{\pi}\sin(x+y+z)\,dx\,dy\,dz = \int_{0}^{\pi}\int_{0}^{\pi}\left(-\cos(x+y+z)\Big|_{x=0}^{\pi}\right)dy\,dz$$

$$= \int_{0}^{\pi}\int_{0}^{\pi}[-\cos(\pi+y+z)+\cos(y+z)]\,dy\,dz$$

$$= \int_{0}^{\pi}\left[-\sin(\pi+y+z)+\sin(y+z)\Big|_{y=0}^{y=\pi}\right]dz$$

$$= \int_{0}^{\pi}[-\sin(2\pi+z)+2\cos(\pi+z)-\sin z]\,dz$$

$$= \cos(2\pi+z)-2\cos(\pi+z)+\cos x\Big|_{0}^{\pi} = -8$$

5.

풀이 $$\int_{1}^{5}\int_{-2x}^{x}\int_{2y}^{x-2} 2\,dz\,dy\,dx = \int_{1}^{5}\int_{-2x}^{x}\left(2z\Big|_{z=2y}^{z=x-2}\right)dy\,dx = 2\int_{1}^{5}\int_{-2x}^{x}(x-2y-2)\,dy\,dx$$

$$= 12\int_{1}^{5}(x^2-x)\,dx = 12\left(\frac{1}{3}x^3-\frac{1}{2}x^2\right)\Bigg|_{1}^{5} = 352$$

7.

풀이 $$\int_{-1}^{1}\int_{x^2}^{x}\int_{0}^{xy} 8xyz\,dz\,dy\,dx = \int_{-1}^{1}\int_{x^2}^{x}\left(4xyz^2\Big|_{z=0}^{z=xy}\right)dy\,dx = \int_{-1}^{1}\int_{x^2}^{x}4x^3y^3\,dy\,dx$$

$$= \int_{-1}^{1}\left(x^3y^4\Big|_{y=x^2}^{y=x}\right)dx = \int_{-1}^{1}(x^7-x^{11})\,dx$$

$$= \frac{1}{8}x^8-\frac{1}{12}x^{12}\Bigg|_{-1}^{1} = 0$$

9.

풀이 $$\int_0^2\int_0^1\int_0^{\sqrt{1-z^2}} xze^y\,dx\,dz\,dy = \int_0^2\int_0^1\left(\frac{1}{2}x^2ze^y\,\Big|_{x=0}^{x=\sqrt{1-z^2}}\right)dz\,dx$$

$$= \frac{1}{2}\int_0^2\int_0^1 (z-z^3)e^y\,dz\,dx$$

$$= \frac{1}{2}\int_0^2\left(-\frac{1}{4}z^4e^y+\frac{1}{2}z^2e^y\,\Big|_{z=0}^{z=1}\right)dy$$

$$= \frac{1}{8}\int_0^2 e^y\,dy = \frac{1}{8}e^y\,\Big|_0^2 = \frac{e^2-1}{8}$$

11.

풀이 $$\int_0^{\pi/3}\int_0^{\ln(\tan z)}\int_{-\infty}^{y} e^{2x}\,dx\,dy\,dz = \int_0^{\pi/3}\int_0^{\ln(\tan z)}\left(\frac{1}{2}e^{2x}\,\Big|_{-\infty}^{y}\right)dy\,dz$$

$$= \frac{1}{2}\int_0^{\pi/3}\int_0^{\ln(\tan z)} e^{2y}\,dy\,dz$$

$$= \frac{1}{2}\int_0^{\pi/3}\left(\frac{1}{2}e^{2y}\,\Big|_0^{\ln(\tan z)}\right)dx$$

$$= \frac{1}{4}\int_0^{\pi/3}\left(e^{2\ln(\tan z)}-1\right)dz$$

$$= \frac{1}{4}\int_0^{\pi/3}\left(\tan^2 z-1\right)dz = \frac{1}{4}(-2z+\tan z)\,\Big|_0^{\pi/3}$$

$$= \frac{\sqrt{3}}{4}-\frac{\pi}{6}$$

13.

풀이 $$\int_0^{\infty}\int_0^{\infty}\frac{1}{(1+x)^2(1+y)^2}\,dx\,dy = \int_0^{\infty}\left(-\frac{1}{(1+x)(1+y)^2}\,\Big|_{x=0}^{x=\infty}\right)dy$$

$$= \int_0^{\infty}\frac{1}{(1+y)^2}\,dy = -\frac{1}{1+y}\,\Big|_0^{\infty} = 1$$

15.

풀이 $\int_0^1\int_{\sqrt{x}}^1\int_0^{1-y} dz\,dy\,dx = \int_0^1\int_{\sqrt{x}}^1 (1-y)\,dy\,dx = \int_0^1\left(\frac{1}{2} - \sqrt{x} + \frac{1}{2}x\right)dx = \frac{1}{12}$

17.

풀이 $\int_0^1\int_{-1}^1\int_0^{y^2} dz\,dy\,dx = \int_0^1\int_{-1}^1 y^2\,dy\,dx = \frac{2}{3}\int_0^1 dx = \frac{2}{3}$

19.

풀이 $\int_0^1\int_0^{4-a-x^2}\int_a^{4-x^2-y} dz\,dy\,dx = \int_0^1\int_0^{4-a-x^2}(4-x^2-y-a)\,dy\,dx$

$= \frac{1}{2}\int_0^1 (4-a-x^2)^2\,dx = \frac{4}{15}$

$\int_0^1 (4-a-x^2)^2\,dx = \int_0^1 \left[(4-a)^2 - 2(4-a)x^2 + x^4\right]dx = (4-a)^2 - \frac{2}{3}(4-a) + \frac{1}{5} = \frac{8}{15}$

$3(4-a)^2 - 2(4-a) - 1 = 0$; $a = \frac{13}{3}$, $a = 3$

Section 8.3 연습문제

1.

풀이 $$\int_0^{\pi/2}\int_0^2 r^2\sin\theta\,dr\,d\theta = \int_0^{\pi/2}\sin\theta\left[\frac{1}{3}r^3\right]_0^2 d\theta = \frac{8}{3}(-\cos\theta)\Big]_0^{\pi/2} = \frac{8}{3}$$

3.

풀이 $$\int_0^{\pi/2}\int_0^{1+\cos\theta} r^2\sin\theta\,dr\,d\theta = \int_0^{\pi/2}\sin\theta\left[\frac{1}{3}r^3\right]_0^{1+\cos\theta} d\theta$$

$$= \frac{1}{3}\int_0^{\pi/2}(1+\cos\theta)^3\sin\theta\,d\theta = \frac{1}{3}\int_1^2 u^3\,du$$

$$= \frac{1}{3}\left[\frac{1}{4}u^4\right]_1^2 = \frac{5}{4}$$

5.

풀이 $$\int_0^{\pi/4}\int_{1-\cos\theta}^{1-\sin\theta} r^2\sin\theta\cos\theta\,dr\,d\theta = \int_0^{\pi/4}\sin\theta\cos\theta\left[\frac{1}{3}r^3\right]_{1-\cos\theta}^{1-\sin\theta} d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4}\sin\theta\cos\theta\left[(1-\sin\theta)^3-(1-\cos\theta)^3\right]d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4}\sin\theta\,(1-\sin\theta)^3\cos\theta\,d\theta + \frac{1}{3}\int_0^{\pi/4}\cos\theta\,(1-\cos\theta)^3(-\sin\theta)\,d\theta$$

$$= \frac{1}{3}\int_0^{1/\sqrt{2}} u\,(1-u)^3\,du + \frac{1}{3}\int_0^{1/\sqrt{2}} u\,(1-u)^3\,du$$

$$= \frac{1}{3}\left(\frac{35-22\sqrt{2}}{80}+\frac{31-22\sqrt{2}}{80}\right) = \frac{11}{120}(3-2\sqrt{2})$$

7.

풀이 $$\int_0^{\pi/4}\int_{1-\cos 2\theta}^{1+\cos 2\theta} r\,dr\,d\theta = \int_0^{\pi/4}\left[\frac{1}{2}r^2\right]_{1-\cos 2\theta}^{1+\cos 2\theta} d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4}\left[(1+\cos 2\theta)^2-(1-\cos 2\theta)^2\right] d\theta$$

$$= 2\int_0^{\pi/4}\cos 2\theta\, d\theta = \sin 2\theta \,\Big|_0^{\pi/4} = 1$$

9.

풀이 $$\int_{\pi/4}^{3\pi/4}\int_{\operatorname{cosec}\theta}^{2\sin\theta} r\,dr\,d\theta = \int_{\pi/4}^{3\pi/4}\left[\frac{1}{2}r^2\right]_{\operatorname{cosec}\theta}^{2\sin\theta} d\theta = \frac{1}{2}\int_{\pi/4}^{3\pi/4}(4\sin^2\theta-\operatorname{cosec}^2\theta)\,d\theta$$

$$= \frac{1}{2}(2\theta - \sin 2\theta + \cot\theta)\Big|_{\pi/4}^{3\pi/4} = \frac{\pi}{2}$$

11.

풀이 $$\int_{-1}^{1}\int_0^{\sqrt{1-x^2}} y\,dy\,dx = \int_0^{\pi}\int_0^1 r^2\sin\theta\,dr\,d\theta = \frac{1}{3}\int_0^{\pi}\sin\theta\,d\theta = \frac{2}{3}$$

13.

풀이 $$\int_0^2\int_0^{\sqrt{2x-x^2}} y^2\,dy\,dx = \int_0^{\pi/2}\int_0^{2\cos\theta} r^3\sin^2\theta\,dr\,d\theta = 4\int_0^{\pi/2}\cos^4\theta\sin^2\theta\,d\theta = \frac{\pi}{8}$$

15.

풀이 $$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy\,dx = \int_0^{2\pi}\int_0^2 r\,dr\,d\theta = \int_0^{2\pi} 2\,d\theta = 4\pi$$

17.

풀이 $$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy = \int_0^{2\pi}\int_0^1 r^3\,dr\,d\theta = \int_0^{2\pi}\frac{1}{4}\,d\theta = \frac{\pi}{2}$$

19.

풀이 $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} \sin(x^2+y^2)\,dy\,dx = \int_0^{\pi}\int_0^2 r\sin r^2\,dr\,d\theta = \int_0^{\pi} \frac{\sin 4}{2}\,d\theta = \frac{\pi \sin 4}{2}$

21.

풀이 원 $r=2$ 외부와 심장형 $r=2(1+\cos\theta)$ 내부로 둘러싸인 영역은

$$D=\{(r,\theta)\,|\,-\pi/2\le\theta\le\pi/2,\ 2\le r<2(1+\cos\theta)\}$$

이고, 이 영역은 기선을 중심으로 대칭이다.

$$\begin{aligned}A&=\iint_D r\,dr\,d\theta = 2\int_0^{\pi/2}\int_2^{2(1+\cos\theta)} r\,dr\,d\theta\\&=2\int_0^{\pi/2}\left(\frac{1}{2}r^2\Big|_{r=2}^{r=2(1+\cos\theta)}\right)d\theta = 4\int_0^{\pi/2}\left[(1+\cos\theta)^2-1\right]d\theta\\&=4\int_0^{\pi/2}(2\cos\theta+\cos^2\theta)\,d\theta = 4\int_0^{\pi/2}\left(\frac{1}{2}+2\cos\theta+\frac{1}{2}\cos 2\theta\right)d\theta\\&=4\left(\frac{1}{2}\theta+2\sin\theta+\frac{1}{4}\sin 2\theta\,\Big|_0^{\pi/2}\right)=8+\pi\end{aligned}$$

23.

풀이 잎 하나의 내부로 둘러싸인 영역은

$$D=\{(r,\theta)\,|\,-\pi/4\le\theta\le\pi/4,\ 0\le r<2\sqrt{\cos 2\theta}\}$$

이고, 이 영역은 기선을 중심으로 대칭이고 동일한 잎이 2개 있다.

$$\begin{aligned}A&=2\iint_D r\,dr\,d\theta = 2\int_{-\pi/4}^{\pi/4}\int_0^{2\sqrt{\cos 2\theta}} r\,dr\,d\theta\\&=4\int_0^{\pi/4}\left(\frac{1}{2}r^2\Big|_{r=0}^{r=2\sqrt{\cos 2\theta}}\right)d\theta = 8\int_0^{\pi/4}\cos 2\theta\,d\theta\\&=4\sin 2\theta\,\Big|_0^{\pi/4}=4\end{aligned}$$

25.

풀이 $u = \dfrac{2x-y}{2}$, $v = \dfrac{y}{2}$이므로 역변환은 $x = u+v$, $y = 2v$이다. 그러므로 야코비안은

$$J = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2$$

이다. 한편 xy평면에서 적분영역 $D = \{(x, y) | 0 \le y \le 4,\ y/2 \le x \le (y/2)+1\}$의 각 변은 다음과 같이 변환된다.

$$\begin{aligned} &x = y/2 \quad \Rightarrow \quad u+v = 2v/2 = v \quad \Rightarrow \quad u = 0 \\ &x = (y/2)+1 \quad \Rightarrow \quad u+v = (2v/2)+1 = v+1 \quad \Rightarrow \quad u = 1 \\ &y = 0 \quad \Rightarrow \quad 2v = 0 \quad \Rightarrow \quad v = 0 \\ &y = 4 \quad \Rightarrow \quad 2v = 4 \quad \Rightarrow \quad v = 2 \end{aligned}$$

$$\int_0^4 \int_{y/2}^{(y/2)+1} \frac{2x-y}{2}\, dx\, dy = \int_0^2 \int_0^1 u|J|\, du\, dv = \int_0^2 \int_0^1 2u\, du\, dv = 2$$

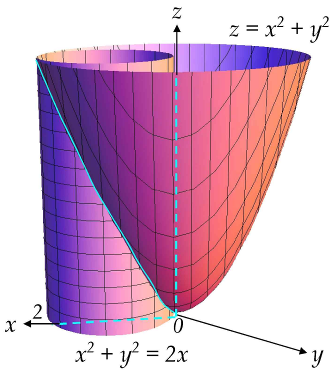
27.

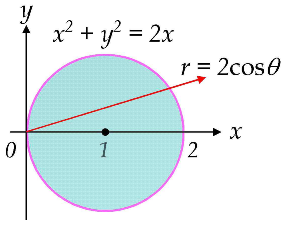
풀이 원기둥 $x^2+y^2 = 2x$의 내부는 $R = \{(x, y) | x^2+y^2 \le 2x\}$이고 이 영역을 극좌표계로 나타내면, $x^2+y^2 = r^2$, $x = r\cos\theta$이므로 영역 R의 경계는 $r^2 = 2r\cos\theta$ 또는 $r = 2\cos\theta$이다. 그러므로 극좌표계로 변환한 영역

$$D = \{(r, \theta) | 0 \le \theta \le \pi,\ 0 \le r \le 2\cos\theta\}$$

이다. 따라서 구하고자 하는 중적분은 다음과 같다.

$$\begin{aligned} \iint_R (x^2+y^2)\, dA &= \iint_D r^3\, dr\, d\theta = \int_0^\pi \int_0^{2\cos\theta} r^3\, dr\, d\theta \\ &= \int_0^\pi \left(\frac{1}{4} r^4 \Big|_{r=0}^{r=2\cos\theta} \right) d\theta = 4\int_0^\pi \cos^4\theta\, d\theta \\ &= 8\int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2} \right)^2 d\theta = 2\int_0^{\pi/2} \left(1 + 2\cos 2\theta + \frac{1+\cos 4\theta}{2} \right) d\theta \\ &= 2\left(\frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta \Big|_0^{\pi/2} \right) = \frac{3\pi}{2} \end{aligned}$$





29.

풀이 $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,\, y)\,dx\,dy = k\int_{0}^{\infty}\int_{0}^{\infty} e^{-2x}e^{-3y}\,dx\,dy$$

$$= k\left(\int_{0}^{\infty} e^{-2x}\,dx\right)\left(\int_{0}^{\infty} e^{-3y}\,dy\right)$$

$$= k\left(\lim_{a\to\infty}\int_{0}^{a} e^{-2x}\,dx\right)\left(\lim_{b\to\infty}\int_{0}^{b} e^{-3y}\,dy\right)$$

$$= k\left(\lim_{a\to\infty} -\frac{1}{2}e^{-2x}\Big|_{0}^{a}\right)\left(\lim_{b\to\infty} -\frac{1}{3}e^{-3y}\Big|_{0}^{b}\right)$$

$$= k\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{k}{6} = 1$$

이므로 $k = 6$이다.

Section 8.4 연습문제

1.

풀이 $$\iiint_R (1+r+\sin\theta)\,dz\,dr\,d\theta = \int_{-\pi}^{\pi}\int_0^2\int_{-1}^{2}(1+r+\sin\theta)\,dz\,dr\,d\theta$$

$$= 3\int_{-\pi}^{\pi}\int_0^2 (1+r+\sin\theta\cos\theta)\,dr\,d\theta$$

$$= 6\int_{-\pi}^{\pi}\int_0^2 (2+\sin\theta\cos\theta)\,dr\,d\theta \;= 24\pi$$

3.

풀이 $$\iiint_R (r^2\sin\theta+2z)\,dz\,dr\,d\theta = \int_0^{\pi}\int_0^2\int_0^2 (r^2\sin\theta+2z)\,dz\,dr\,d\theta$$

$$= 2\int_0^{\pi}\int_0^2 (2+r^2\sin\theta)\,dr\,d\theta$$

$$= 8\int_0^{\pi}\left(1+\frac{2\sin\theta}{3}\right)d\theta = 8\pi+\frac{32}{3}$$

5.

풀이 $$\iiint_R 3\rho^2\sin\phi\;d\rho\,d\phi\,d\theta = \int_0^{\pi}\int_0^{\pi/3}\int_{\sec\phi}^{2} 3\rho^2\sin\phi\;d\rho\,d\phi\,d\theta$$

$$= \int_0^{\pi}\int_0^{\pi/3}(8\sin\phi-\sec^2\phi\tan\phi)\,d\phi\,d\theta = \int_0^{\pi}\frac{5}{2}\,d\theta = \frac{5\pi}{2}$$

7.

풀이 $$\iiint_R \sqrt{x^2+y^2}\;dV = \int_0^{2\pi}\int_0^2\int_0^{9-r\cos\theta-2r\sin\theta} r^2\;dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 r^2(9-r\cos\theta-2r\sin\theta)\,dr\,d\theta$$

$$= 4\int_0^{2\pi}(6-\cos\theta-2\sin\theta)\,d\theta \;= 48\pi$$

9.

풀이 $V=\int_0^{\pi/2}\int_0^2\int_0^4 r\,dz\,dr\,d\theta=4\int_0^{\pi/2}\int_0^2 r\ dr\,d\theta=8\int_0^{\pi/2}d\theta\ =4\pi$

11.

풀이 $V=\int_0^{\pi/2}\int_0^{2\cos\theta}\int_0^{4-r^2} r\,dz\,dr\,d\theta=\int_0^{\pi/2}\int_0^{2\cos\theta} r\,(4-r^2)\,dr\,d\theta$

$=4\int_0^{\pi/2}(2\cos^2\theta-\cos^4\theta)\,d\theta\ =\dfrac{5\pi}{4}$

13.

풀이 $V=\int_0^{\pi}\int_1^{1+\cos\theta}\int_0^2 r\,dz\,dr\,d\theta=\int_0^{\pi}\int_1^{1+\cos\theta}2r\,dr\,d\theta=\int_0^{\pi}[(1+\cos\theta)^2-1]\,d\theta\ =\dfrac{\pi}{2}$

15.

풀이 $\iiint_R dV=\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\phi}\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

$=\dfrac{1}{3}\int_0^{2\pi}\int_0^{\pi/4}\cos^3\phi\sin\phi\ d\phi\,d\theta\ =\dfrac{1}{12}\int_0^{2\pi}\dfrac{3}{4}\,d\theta\ \ =\dfrac{\pi}{8}$

17.

풀이 $\iiint_R dV=\int_0^{2\pi}\int_0^{\pi/2}\int_0^3\rho^2\sin\phi\,d\rho\,d\theta\,d\phi=9\int_0^{2\pi}\int_0^{\pi/2}\sin\phi\ d\phi\,d\theta\ =9\int_0^{2\pi}d\theta=18\pi$

19.

풀이 $\iiint_R dV=\int_0^{2\pi}\int_0^{\pi/3}\int_0^{1+\cos\phi}\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

$=\dfrac{1}{3}\int_0^{2\pi}\int_0^{\pi/3}(1+\cos\phi)^3\sin\phi\ d\phi\,d\theta=\dfrac{1}{3}\int_0^{2\pi}\dfrac{175}{64}\,d\theta\ =\dfrac{175\pi}{96}$