Chapter 01. 연습문제 해답

1.1.

[정답]

(a), (c), (f)

1.2.

[정답]

(a) $x=3,\ y=4$

(b) $x=-4,\ y=0,\ z=2$

(c) $x=3,\ y=1,\ z=0$

(d) $x_1=7-2t+6s,\ x_2=-3-t-3s,\ x_3=t,\ x_4=s$

(e) $x=7t/4,\ y=-t/2,\ z=-15t/4,\ w=t$

(f) $z=1,\ y=1,\ x=2,\ w=4,\ v=8$

(g) $x=1-2t-s,\ y=t,\ z=2-s,\ w=s$

(h) $x_1=1/2,\ x_2=-1/2,\ x_3=0,\ x_4=-1$

(i) $x_1=-2,\ x_2=-1,\ x_3=0,\ x_4=0$

(j) $v=1-t-2s,\ w=1-t,\ x=t,\ y=s,\ z=0$

1.3.

[정답]

(a) $\begin{bmatrix} 1 & -3 & 0 & -3 & 0 & -4 \\ 0 & 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & \frac{1}{2} & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

1.4.

[정답]

$a=\frac{16}{5}, b=-\frac{13}{10}, c=-\frac{31}{10}$

1.5.

[정답]

$\sin\alpha = -1, \cos\beta = 1, \tan\gamma = 0$이므로

$\therefore \alpha = \frac{3\pi}{2},\ \beta = 0, \pi,\ \gamma = 0$

1.6.

[정답]

해가 없거나 무수히 많은 해를 가진다.

1.7.

[정답]

$y = \frac{3}{2}x^2 - \frac{11}{2}x + 3$

1.8.

[정답]

$x = 1,\ y = 1,\ z = 1$
$x = 1,\ y = 2,\ z = 2$
$x = 1,\ y = 2,\ z = 3$

1.9.

[정답] $a = -3$

1.10.

[정답]

무수히 많은 해를 가진다.

1.11.

[정답]

(a) $a = 3$

(b) $a \neq 2, 3$

(c) $a = 2$

1.12

[정답]

마지막 행의 결과 마지막 변수의 값이 0(영)이고 이를 윗식에 대입하여 같은 방법으로 풀이하면 자명한 해만을 갖는다.

Chapter 02 연습문제 해답

2.1.

[정답]

(a) $a_{12} = -2,\ a_{22} = -3,\ a_{23} = 4$

(b) $b_{11} = 2,\ b_{31} = 5$

(c) $c_{13} = 2,\ c_{31} = 7,\ c_{33} = -1$

(d) $6,\ 3,\ -1$

2.2.

[정답]

$x = 0,\ y = -3,\ z = -1,\ w = -1$

2.3.

[정답]

(a) $A^T = \begin{bmatrix} 1 & -1 & -1 \\ 5 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

(b) $\mathrm{tr}(A) = 5$

(c) $(A+B)^T = \begin{bmatrix} 4 & 1 & -4 \\ 8 & -1 & 3 \\ 5 & 2 & 6 \end{bmatrix}$

(d) $\mathrm{tr}(A+B) = 9$

2.4.

[증명 생략]

2.5.

[증명 생략]

(a) $\mathrm{tr}(AB) = -7 = \mathrm{tr}(BA)$

2.6.

[정답]

(a) $\begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{21} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{9} & \frac{2}{9} \\ -\frac{4}{27} & \frac{1}{27} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{19}{18} & -\frac{7}{6} & -\frac{11}{18} \\ -\frac{1}{9} & \frac{1}{3} & \frac{2}{9} \\ -\frac{7}{18} & \frac{1}{6} & \frac{5}{18} \end{bmatrix}$ (e) $\begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{17}{3} & \frac{22}{3} & -\frac{7}{3} \\ \frac{4}{3} & \frac{5}{3} & -\frac{2}{3} \end{bmatrix}$ (f) 비가역

(g) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$ (h) $\begin{bmatrix} \frac{17}{7} & -2 & -\frac{3}{7} & -\frac{6}{7} \\ \frac{16}{7} & -1 & \frac{5}{7} & -\frac{11}{7} \\ \frac{11}{7} & -1 & \frac{3}{7} & -\frac{8}{7} \\ -\frac{9}{7} & 1 & \frac{2}{7} & \frac{4}{7} \end{bmatrix}$

(i) 비가역 (j) $\begin{bmatrix} \frac{1}{k} & 0 & 0 & 0 \\ -\frac{1}{k^2} & \frac{1}{k} & 0 & 0 \\ \frac{1}{k^3} & -\frac{1}{k^2} & \frac{1}{k} & 0 \\ -\frac{1}{k^4} & \frac{1}{k^3} & -\frac{1}{k^2} & \frac{1}{k} \end{bmatrix}$

2.7.

[정답]

$k = -3$

2.8.

[정답]

(a) $x_1 = 0,\ x_2 = 0,\ x_3 = 0$

(b) $x_1 = 2,\ x_2 = 1,\ x_3 = -1$

2.9.

[정답]

$$A = \begin{bmatrix} -2 & \frac{7}{2} & -\frac{1}{2} & 0 \\ \frac{7}{2} & 1 & 1 & \frac{3}{2} \\ -\frac{1}{2} & 1 & 1 & 2 \\ 0 & \frac{3}{2} & 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{5}{2} & 0 \\ -\frac{5}{2} & 0 & -3 & \frac{3}{2} \\ -\frac{5}{2} & 3 & 0 & 0 \\ 0 & -\frac{3}{2} & 0 & 0 \end{bmatrix}$$

2.10.

[정답]

$x \neq -2,\ 1,\ 4$

2.11.

[정답]

$a=0,\ b=3$

2.12.

[정답]

$a=-1,\ 3$

2.13.

[정답]

$a+b-c\neq 0$

2.14.

[정답]

$$D^{-1}=\begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{8} & 0 & 0 \\ 0 & 0 & \frac{1}{7} & 0 \\ 0 & 0 & 0 & -\frac{1}{9} \end{bmatrix}$$

2.15.

[정답]

$$A^3=0,\ \begin{bmatrix} 1 & r & r^2+s \\ 0 & 1 & r \\ 0 & 0 & 1 \end{bmatrix}$$

2.16.

[증명 생략]

힌트 : $(A+B)^2=A^2+AB+BA+B^2$

2.17.

[정답]

(a) $A^{-1}=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 5 & 2 & 1 \end{bmatrix}$

(b) $A^{-1}=\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

2.18

[증명 생략]

2.19.

[증명 생략]

2.20.

[증명 생략]

2.21.

[증명 생략]

2.22.

[정답]

$$A^n = \begin{bmatrix} \frac{1}{3^n} & \frac{n}{3^{n-1}}\alpha \\ 0 & \frac{1}{3^{n-1}} \end{bmatrix} \quad \therefore \lim_{n\to\infty} A^n = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2.23.

[증명 생략]

2.24.

[정답]

$$X = (I - A)^{-1}B = \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 3 & 3 \end{bmatrix}$$

2.25.

[증명 생략]

Chapter 03 연습문제 해답

3.1.

[정답]

(a) 39 (b) 0 (c) 8

(d) $-abc$ (e) 39 (f) 4

3.2.

[정답]

$r = 0,\ \frac{1}{3},\ 4$

3.3.

[정답]

$\frac{3}{4} \pm \frac{\sqrt{33}}{4}$

3.4.

[증명 생략]

3.5.

[정답]

$a = 3,\ -2$

3.6.

[정답]

$\lambda = 1,\ 2,\ 2$

3.7.

[정답]

$2^6 = 64$

3.8.

[정답]

(a) $adj(A) = \begin{bmatrix} 18 & 10 & 6 \\ -17 & 1 & 10 \\ 6 & -28 & 2 \end{bmatrix}$

(b) $adj(B) = \begin{bmatrix} -5 & 1 & -2 \\ 9 & -6 & -9 \\ -6 & -3 & 6 \end{bmatrix}$

3.9.

[정답]

(a) $x_2 = 13$

(b) $x_2 = \frac{9}{37}$

3.10.

[정답]

$a = -8, 5$

$a = 5$일 때 $(-t, t, t)$, $a = -8$일 때 $(t, 2t, t)$

3.11.

[증명 생략]

3.12.

[증명 생략]

3.13.

[증명 생략]

3.14.

[증명 생략]

3.15.

[증명 생략]

3.16.

[증명 생략]

3.17.

[증명 생략]

3.18.

[증명 생략]

3.19.

[정답]

$p(x) = 1 - \frac{421}{36}x + \frac{251}{72}x^2 + \frac{215}{72}x^3 - \frac{7}{9}x^4$

Chapter 04 연습문제 해답

4.1.

[정답]

(a) 4개 (b) 4개 (c) 4개 (d) 3개 (e) $\sqrt{17}$

4.2.

[정답]

(a) $\mathbf{a} = \frac{1}{\sqrt{33}}(2\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$

(b) $\mathbf{a} = \frac{1}{\sqrt{21}}(4, -1, 2)$

(c) $\mathbf{a} = \frac{1}{\sqrt{10}}(3\mathbf{i} + \mathbf{j})$

(d) $\mathbf{a} = \frac{1}{\sqrt{30}}(2, -1, 5)$

4.3.

[정답]

(a) $(-14, -5, -36)$

(b) $(21, -9, -1)$

4.4.

[정답]

(a) $(-22, -24, -116)$

(b) $(-4, 36, -26)$

(c) $(0, 198, -132)$

(d) -198

4.5.

[정답]

(a) $\frac{4}{5}$

(b) 0

(c) $\frac{667}{940}$

(d) $\frac{649}{701}$

4.6.

[정답] 49

4.7.

[정답]

(a) $\frac{1020}{743}$

(b) $\frac{769}{3524}$

4.8.

[정답]

$24x + 12y + 8z - 24 = 0$

4.9.

[정답]

$$\begin{cases} x = -14t - 13 \\ y = -t - 17 \\ z = t \end{cases}$$

4.10.

[정답]

$-6x + 2y - 3z = 0$

4.11.

[정답]

$(\frac{13}{9}, \frac{14}{9}, \frac{2}{9})$

4.12.

[정답]

$\frac{5817}{379}$

4.13.

[정답]

$\frac{51}{\sqrt{54}}$

4.14

[증명 생략]

4.15.

[증명 생략]

4.16.

[증명 생략]

4.17.

[증명 생략]

4.18.

[증명 생략]

4.19.

[증명 생략]

4.20.

[증명 생략]

4.21.

[증명 생략]

4.22.

[증명 생략]

Chapter 05. 연습문제 해답

5.1.

[정답]

(a) 부분공간
(b) 부분공간 아님
(c) 부분공간 아님
(d) 부분공간 아님

5.2.

[정답]

(a) 부분공간 아님
(b) 부분공간
(c) 부분공간
(d) 부분공간 아님

5.3.

[정답]

(a) 일차독립
(b) 일차독립
(c) 일차독립
(d) 일차독립

5.4.

[정답]

(c),(e)

5.5.

[정답] 일차독립이고 생성하므로 기저이다.

5.6.

[증명 생략]

5.7.

[정답]

(a) 일차독립
(b) 일차독립
(c) 일차종속

5.8.

[정답]

$\{(1,0,1,0),(0,1,-1,0),(0,0,1,0),(0,0,0,1)\}$

5.9.

[정답]

3차원

5.10.

[정답]

$$\left\{\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix},\ \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix},\ \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}\right\}$$

5.11.

[정답]

$$\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\ \begin{bmatrix}0\\1\\0\\-3\end{bmatrix},\ \begin{bmatrix}0\\0\\1\\4\end{bmatrix}$$

5.12.

[정답]

(a) $\text{rank}(A)(2)+\text{nullity}(A)(2)=4$

(b) $\text{rank}(A)(4)+\text{nullity}(A)(1)=5$

5.13.

[정답]

$\{(10,11,7)\}$, 차원=1

5.14.

[정답]

(a) $[\mathbf{x}]_T=\begin{bmatrix}\frac{4}{3}\\\frac{1}{3}\\-\frac{8}{3}\end{bmatrix}$, $[\mathbf{y}]_T=\begin{bmatrix}1\\1\\-3\end{bmatrix}$

(b) $\begin{bmatrix}\frac{1}{8}&\frac{5}{8}&-\frac{3}{4}\\\frac{7}{8}&\frac{11}{8}&\frac{3}{4}\\\frac{3}{8}&-\frac{1}{8}&-\frac{1}{4}\end{bmatrix}$

(c) $[\mathbf{x}]_S=\begin{bmatrix}\frac{1}{2}\\-1\\\frac{3}{2}\end{bmatrix}$, $[\mathbf{y}]_T=\begin{bmatrix}0\\3\\-2\end{bmatrix}$

(d) $\begin{bmatrix}\frac{1}{8}&\frac{5}{8}&-\frac{3}{4}\\\frac{7}{8}&\frac{11}{8}&\frac{3}{4}\\\frac{3}{8}&-\frac{1}{8}&-\frac{1}{4}\end{bmatrix}^{-1}$

5.15.

[정답]

(a) 직교

(b) 직교 아님

5.16.

[정답]

$$\text{(a)}\quad T=\left\{\mathbf{y}_1=\begin{bmatrix}\frac{929}{3476}\\ \frac{929}{1738}\\ \frac{809}{1009}\end{bmatrix},\ \mathbf{y}_2=\begin{bmatrix}-\frac{780}{1351}\\ -\frac{780}{1351}\\ \frac{780}{1351}\end{bmatrix},\ \mathbf{y}_3=\begin{bmatrix}-\frac{753}{976}\\ \frac{753}{1220}\\ -\frac{649}{4206}\end{bmatrix}\right\}$$

$$\text{(b)}\quad T=\left\{\mathbf{y}_1=\begin{bmatrix}\frac{379}{1257}\\ \frac{379}{419}\\ -\frac{379}{1257}\end{bmatrix},\ \mathbf{y}_2=\begin{bmatrix}\frac{1531}{1636}\\ -\frac{434}{1971}\\ \frac{657}{2387}\end{bmatrix},\ \mathbf{y}_3=\begin{bmatrix}-\frac{461}{2525}\\ \frac{505}{1383}\\ \frac{461}{505}\end{bmatrix}\right\}$$

5.17.

[정답]

$$\text{(a)}\,Q=\begin{bmatrix}-\frac{747}{4174} & -\frac{889}{1136} & -\frac{963}{1615}\\ \frac{498}{2087} & -\frac{1400}{2249} & \frac{963}{1292}\\ -\frac{671}{703} & -\frac{20}{2249} & \frac{963}{3230}\end{bmatrix},\ R=\begin{bmatrix}-\frac{4174}{249} & \frac{3638}{231} & \frac{3638}{693}\\ 0 & -\frac{1347}{374} & -\frac{449}{374}\\ 0 & 0 & *\end{bmatrix}$$

$$\text{(b)}\,Q=\begin{bmatrix}-\frac{881}{2158} & \frac{780}{1351} & \frac{985}{1393}\\ -\frac{881}{1079} & -\frac{780}{1351} & *\\ \frac{881}{2185} & -\frac{780}{1351} & \frac{985}{1393}\end{bmatrix},\ R=\begin{bmatrix}-\frac{2158}{881} & 0 & -\frac{1079}{881}\\ 0 & -\frac{1347}{374} & 0\\ 0 & 0 & -\frac{985}{1393}\end{bmatrix}$$

5.18.

[정답]

$$T=\left\{\mathrm{y}_1=\begin{bmatrix}\frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\end{bmatrix},\ \mathrm{y}_2=\begin{bmatrix}\frac{1}{\sqrt{6}}\\ \frac{1}{\sqrt{6}}\\ -\sqrt{\frac{2}{3}}\end{bmatrix},\ \mathrm{y}_3=\begin{bmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\\ 0\end{bmatrix}\right\}$$

$\mathrm{v}=(\mathrm{v}\cdot\mathrm{y}_1)\mathrm{y}_1+(\mathrm{v}\cdot\mathrm{y}_2)\mathrm{y}_2+(\mathrm{v}\cdot\mathrm{y}_3)\mathrm{y}_3$

5.19.

[정답]

$$\begin{bmatrix}\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3}\\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{6}}{3} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6}\end{bmatrix}$$

5.20.

[정답]

$a=0, b=-\sqrt{\frac{2}{3}}, c=\sqrt{\frac{1}{3}}$

5.21.

[증명 생략]

5.22.

[증명 생략]

Chapter 06 연습문제 해답

6.1.

[정답]

(a) $\left\{\begin{bmatrix} -\frac{257}{296} \\ \frac{257}{518} \end{bmatrix}, \begin{bmatrix} -\frac{985}{1393} \\ -\frac{985}{1393} \end{bmatrix}\right\}, \ -2, 9$

(b) $\left\{\begin{bmatrix} -\frac{2584}{2889} \\ \frac{1292}{2889} \end{bmatrix}, \begin{bmatrix} -\frac{985}{1393} \\ -\frac{985}{1393} \end{bmatrix}\right\}, \ -4, 2$

(c) $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ * \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ * \end{bmatrix}\right\}, \ 1, 1, 1$

(d) $\left\{\begin{bmatrix} \frac{209}{362} \\ -\frac{209}{362} \\ -\frac{877}{1519} \end{bmatrix}, \begin{bmatrix} -\frac{571}{989} \\ \frac{571}{989} \\ \frac{209}{362} \end{bmatrix}, \begin{bmatrix} -\frac{571}{989} \\ \frac{571}{989} \\ \frac{209}{362} \end{bmatrix}\right\}, \frac{115262}{115261}, \frac{230521}{230522}, \frac{230521}{230522}$

(e) $\left\{\begin{bmatrix} -\frac{803}{839} \\ \frac{295}{1018} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -\frac{295}{1018} \\ -\frac{803}{839} \\ 0 \end{bmatrix}\right\}, \ -\frac{1069}{120}, -1, \frac{2498}{1309}$

(f) $\left\{\begin{bmatrix} 0 \\ \frac{985}{1393} \\ 0 \\ -\frac{985}{1939} \end{bmatrix}, \begin{bmatrix} -\frac{521}{991} \\ 0 \\ \frac{3275}{4379} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{985}{1393} \\ 0 \\ \frac{985}{1393} \end{bmatrix}, \begin{bmatrix} -\frac{3725}{4379} \\ 0 \\ -\frac{521}{991} \\ 0 \end{bmatrix}\right\}, -1, -\frac{610}{987}, 1, \frac{1597}{987}$

6.2.

[정답]

(a) 고윳값 2에 대한 고유공간의 기저 $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$,

고윳값 3에 대한 고유공간의 기저 $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(b) 고윳값 -1에 대한 고유공간의 기저 $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$,

(c) 고윳값 1에 대한 고유공간의 기저 $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$,

고윳값 5에 대한 고유공간의 기저 $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$,

(d) 고윳값 1에 대한 고유공간의 기저 $\begin{bmatrix} 4 \\ 4 \\ 3 \\ 52 \end{bmatrix}$,

고윳값 2에 대한 고유공간의 기저 $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$,

고윳값 3에 대한 고유공간의 기저 $\begin{bmatrix} 0 \\ -3 \\ 1 \\ 17 \end{bmatrix}$,

고윳값 9에 대한 고유공간의 기저 $\begin{bmatrix} 0 \\ 0 \\ 7 \\ 8 \end{bmatrix}$,

6.3.

[정답]

(b) $P_A(\lambda) = \lambda^3 + 5\lambda^2 + 3\lambda - 9$이므로

$A^5 = -86A^2 - 102A + 198I,\ A^{-1} = \frac{1}{9}(A^2 + 5A + 3I)$

6.4.

[증명 생략]

6.5.

[증명 생략]

6.6.

[증명 생략]

6.7.

[정답]

$$P=\begin{bmatrix} \frac{985}{1393} & \frac{684}{721} \\ \frac{985}{1393} & \frac{228}{721} \end{bmatrix}$$

6.8.

[증명 생략]

6.9.

[정답]

고윳값 $0, \frac{1}{4}, 1$

6.10.

[증명 생략]

6.11.

[정답]

(a) $P^{-1}\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}P=\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$

(b) $P^{-1}AP=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

(c) $P^{-1}AP=\begin{bmatrix} -\frac{408}{985} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{985}{408} \end{bmatrix}$

(d) $P^{-1}AP=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 16 \end{bmatrix}$

6.12.

[증명 생략]

6.13.

[정답]

$(-1)^{10}, 7^{10}, 9^{10}, (-8)^{10}, 10^{10}$

6.14.

[증명 생략]

6.15.

[정답]

$y_1(t)=0,\ y_2(t)=-e^{3t},\ y_3(t)=e^{3t}$

Chapter 07 연습문제 해답

7.1.

[증명 생략]

7.2.

[증명 생략]

7.3.

[증명 생략]

7.4.

[정답]

(a) $L(1,\ 2,\ 3) = (3,\ -13)$

(b) $L(1,\ -2,\ 3) = (11,\ -21)$

(c) $\ker L = \{t(-2,3,4) | t \in R\}$

(d) $\mathrm{Im} L = R^2$

7.5.

[정답]

(a) $L(x^2 - 7x + 4) = -26x^2 + 7x - 9$

(b) $L(x^2 - 3x + \sqrt{2}) = -\frac{1572}{199}x^2 - 3x + \frac{4552}{1189}$

(c) $\ker L = \{0\}$

(d) $\mathrm{Im} L = P_2$

7.6.

[정답]

(a) $\begin{cases} x_1 = -w_1 + 2w_2 \\ x_2 = w_1 - w_2 \end{cases}$

(b) $\begin{cases} x_1 = w_1 - 2w_2 + 4w_3 \\ x_2 = -w_1 + 2w_2 - 3w_3 \\ x_3 = -w_1 + 3w_2 - 5w_3 \end{cases}$

7.7.

[정답] $(\frac{3+\sqrt{3}}{4}, \frac{1+\sqrt{3}}{4})$

7.8.

[정답]

(a) $\mathrm{rank}(L) = 3$

(b) $\mathrm{nullity}(L) = 1$

7.9.

[정답]

(a) $\begin{bmatrix} -9 & 0 \\ 0 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & -1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

7.10.

[정답]

7.11.

[정답]

$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, $L(\mathbf{x}) = \begin{bmatrix} k \\ 2k \end{bmatrix}$

7.12.

[증명 생략]

7.13.

[증명 생략]

7.14.

[정답]

(a) $L(tA+sB) = tA+sB-(tA+sB)^T = t(A-A^T)+s(B-B^T) = tL(A)+sL(B)$

(b) $\{A \in M_n \mid A = A^T\}$

(c) $\{A \in M_n \mid A_{ii} = 0, i = 1,\ldots n\}$

7.15.

[증명 생략]

Chpater 08 연습문제 해답

8.1.

[정답]

(a) $z_1 + z_2 = -1 + 4i$

(b) $z_1 - z_2 = 5 - 5i$

(c) $z_1 z_2 = -2 + 11i$

(d) $\frac{z_1}{z_2} = -\frac{2}{5} - \frac{1}{5}i$

(e) $-3z_1\overline{z_2} = 30 + 15i$

(f) $\frac{2\overline{z_1}}{z_2} = -\frac{4}{25} - \frac{22}{25}i$

8.2.

[정답]

(a) $(1 + i + i^2 + i^3)^{2020} = 0$

(b) $(1 - 3i)^4 = 28 + 96i$

(c) $\left(\frac{1}{i}\right)^{2100} = 1$

8.3.

[정답]

$x_1 = i, x_2 = 0, x_3 = -i$

8.4.

[정답]

(a) $3i = 3(\cos\frac{355}{226} + i\sin\frac{355}{226})$

(b) $-8 = 8(\cos\pi + i\sin\pi)$

(c) $7 + 7i = 7\sqrt{2}\,(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

(d) $2\sqrt{3} - 2i = \frac{1351}{390}(\cos(-\frac{501}{814}) + i\sin(-\frac{501}{814}))$

8.5.

[정답]

(a) $\|\mathbf{u}\| = \frac{721}{228}$

(b) $\|\mathbf{v}\| = \frac{2251}{418}$

(c) $\|\mathbf{u}+\mathbf{v}\| = \frac{1941}{296}$

(d) $\|\mathbf{u}\| + \|\mathbf{v}\| = \frac{1171}{137}$

8.6.

[정답]

(a) $2+5i$ (b) 0

8.7.

[정답]

(a) $7-5i$ (b) $-1-i$

8.8.

[정답]

에르미트 행렬 : (a), (b), (g), (h)

유니타리 행렬 : (d), (e), (g), (i)

8.9.

[정답]

(a) $U=\begin{bmatrix} \frac{1157}{2069}-\frac{536}{19171}i & \frac{1475}{2113}-\frac{1349}{3865}i \\ -\frac{1134}{1453} & \frac{367}{587} \end{bmatrix}$, $D=\begin{bmatrix} -\frac{987}{551} & 0 \\ 0 & \frac{1538}{551} \end{bmatrix}$

(b) $U=\begin{bmatrix} 0 & 0 & 1 \\ \frac{881}{2158}+\frac{881}{2158}i & \frac{780}{1351}+\frac{780}{1351}i & 0 \\ -\frac{881}{1079} & \frac{780}{1351} & 0 \end{bmatrix}$, $D=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

8.10.

[증명 생략]

8.11.

[정답]

(a) $\|1+ix\|=\sqrt{\frac{4}{3}}$

(b) $<x,\ 1+ix>=\frac{1}{2}-\frac{1}{3}i$

(c) $<1+ix,\ x>=\frac{1}{2}+\frac{1}{3}i$

8.12.

[정답]

$T=\{\mathbf{v}_1=(i,\ 0,\ 0),\ \mathbf{u}_2=(0,\ 0,\ i),\ \mathbf{u}_3=(0,\ -i,\ 0)\}$

8.13.

[증명 생략]

8.14.
[증명 생략]

8.15.
[증명 생략]

8.16.
[증명 생략]

8.17.
[증명 생략]

8.18.
[증명 생략]

8.19.
[증명 생략]

8.20.
[증명 생략]

8.21.
[증명 생략]